EE3124 Assignment 1 (Solution)

Name: **Student No.:**

Q1 - What are Electric machines? And briefly list the machine requirements for modern electric vehicles.

Solution

An electrical machine is a device which converts mechanical energy into electrical energy or vice versa.

Electrical machines also include transformers, which do not actually make conversion between mechanical and electrical form but they convert AC current from one voltage level to another voltage level.

EV machine requirements are much more stringent than that of industrial machines: (any 4 or more items can get full points)

High torque and power densities.

High torque capability for electric launching and hill climbing.

High efficiency over wide torque and speed ranges.

Wide-speed operating range for low-speed creeping and high-speed cruising.

Wide constant-power operating capability.

High intermittent overload capability for overtaking.

High reliability and robustness for vehicular environment.

Publicly acceptable cost.

– What is magnetizing intensity? What is magnetic flux density? How are they related? (5 pts)

Solution (any reasonable answers can get full points)

Magnetic Flux Density and Permeability

- The magnetic field intensity H is in a sense a measure of the effort that a current is putting into the establishment of a magnetic field. The strength of the magnetic field flux produced in a core also depends on the material of the core.
- The relationship between the magnetic field intensity H and the resulting magnetic flux density B produced within a material is given by

$$B = \mu H$$

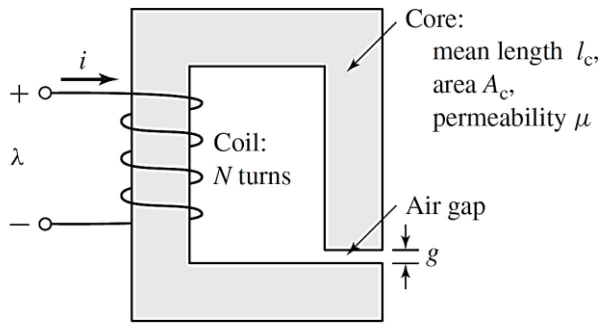
Q2 - An inductor of the form of the following figure has dimensions

Cross-sectional area $A_c = 3.8\ \text{cm}^2$

Mean core length $l_c = 19$ cm

$N = 122$ turns

Assuming a core permeability of $\mu = 3240\,\mu_0$ and neglecting the effects of leakage flux and fringing fields, calculate the air-gap length required to achieve an inductance of 6.0 mH.



Solution

$$g = \frac{\mu_0 N^2 A_c}{L} - \left(\frac{\mu_0}{\mu}\right) l_c = 1.126\ \text{mm}$$

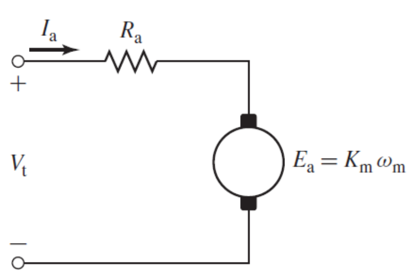
Q3 – A permanent-magnet dc motor is known to have an armature resistance of 1.13 Ω. When operated at no load from a dc source of 52 V, it is observed to rotate at a speed of 2000 r/min and to draw a current of 1.30 A. Find

(a) the torque constant K_m ($K_e\phi$)

(b) the no-load rotational losses of the motor, and

(c) the power output of the motor when it is operating at 1800 r/min from a 50-V source.

Solution



a. From the equivalent circuit, the generated voltage E_a can be found as

$$E_a = V_t - I_a R_a$$

$$= 52\text{-}1.30 \text{ x } 1.13 = 50.531 \text{ V}$$

At a speed of 2000 r/min,

$$\omega_m = \left(\frac{2000 \text{ r}}{\text{min}}\right) \times \left(\frac{2\pi \text{ rad}}{\text{r}}\right) \times \left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$$

$$= 209.4 \text{rad/sec}$$

Therefore,

$$K_m = \frac{E_a}{\omega_m} = \frac{50.531}{209.4} = 0.241 \text{V/(rad/sec)}$$

b. At no load, all the power supplied to the generated voltage E_a is used to supply rotational losses. Therefore

$$\text{Rotational losses } = E_a I_a = 50.531 \text{x} 1.3 = 65.7 \text{ W}$$

c. At 1700 r/min,

$$\omega_m = 1800\left(\frac{2\pi}{60}\right) = 188.5 \text{rad/sec}$$

and

$$E_a = K_m \omega_m = 0.241 \text{x} 188.5 = 45.43 \text{ V}$$

The input current can now be found as

$$I_a = \frac{V_t - E_a}{R_a} = \frac{50\text{-}45.43}{1.13} = 4.04 \text{ A}$$

The electromagnetic power can be calculated as

$$P_{mech} = E_a I_a = 45.43 \text{x} 4.04 = 183.7 \text{ W}$$

Assuming the rotational losses to be constant at their no-load value (certainly an approximation), the output shaft power can be calculated:

$$P_{shaft} = P_{mech} - \text{rotational losses} = 118 \text{ W}$$

Q4 - A shunt-connected, 75-kW, 250-V dc motor has an armature resistance of 45 mΩ and a field resistance of 185 Ω. When operated at 250 V, its no-load speed is 1850 r/min.

a) The motor is operating under load at a terminal voltage of 250 V and a terminal current of 290 A. Calculate (i) the motor speed in r/min, (ii) the load power in kW, and (iii) the load torque in Nm.

b) Assuming the load torque remains constant, calculated in part (a), calculate (i) the motor speed and (ii) the terminal current if the terminal voltage is reduced to 200 V.

c) Repeat part (b) if the load torque of part (a) varies with the square of the speed.

Solution

If we write $E_a = \omega_m K_a I_f$, we can find K_a. At no load given no-load condition with $E_a = V_a = 250$ V, $I_f = 250/185 = 1.35$ A and $\omega_m = 1850 \times \pi/30 = 193.7$ rad/sec. Thus,

$$K_a = 0.956$$

Part (a): Here $I_f = 1.35$ A, $I_L = 290$ A and $I_a = I_L - I_f = 288.6$ A. Thus

$$E_a = V_a - IaR_a = 237 \text{ V}$$

(i)

$$\omega_m = \frac{E_a}{K_a I_f} = 183.6 \text{ rad/sec}$$

corresponding to a speed of 1754 r/min.

(ii)

$$P_{load} = E_a I_a = 68.40 \text{ kW}$$

(iii)

$$T_{load} = \frac{P_{load}}{\omega_m} = 372.5 \text{ N} \cdot \text{m}$$

Part (b): If the terminal voltage is 200 V, $I_f = 200/185 = 1.08$ A.

$$T_{load} = \frac{E_a I_a}{\omega_m} = (V_a - E_a)\left(\frac{K_a I_f}{R_a}\right)$$

and thus

$$E_a = V_a - \frac{T_{load} R_a}{K_a I_f} = 183.5 \text{ V}$$

from which (i)

$$\omega_m = \frac{E_a}{K_a I_f} = 177.7 \text{ rad/sec}$$

corresponding to a speed of 1697 r/min.

(ii)

$$I_L = \frac{V_a - E_a}{R_a} + I_f = 367.8 \text{ A}$$

Part (c): Let $T_0 = 372.5$ N·m and $\omega_{m0} = 183.6$ rad/sec. We can write

$$T_0\left(\frac{\omega_m}{\omega_{m0}}\right)^2 = k_a I_f I_a = K_a I_f \frac{(V_a - K_a \omega_m I_f)}{R_a}$$

from which we find that $\omega_m = 178.8$ rad/sec corresponding to 1707 r/min.

$$I_L = \frac{V_a - \omega_m K_a I_f}{R_a} + I_f = 344.6 \text{ A}$$

Q5 - A three-phase Y-connected 60-Hz two-pole synchronous machine has a stator with 5000 turns of wire per phase. What rotor flux would be required to produce a terminal (line-to-line) voltage of 13.2 kV?

Solution

The phase voltage of this machine should be $V_\phi = V_L / \sqrt{3} = 7621\ \text{V}$. The induced voltage per phase in this machine (which is equal to V_ϕ at no-load conditions) is given by the equation

$$E_A = \sqrt{2}\pi N_C \phi f$$

so

$$\phi = \frac{E_A}{\sqrt{2}\pi N_C f} = \frac{7621\ \text{V}}{\sqrt{2}\pi (5000\ \text{t})(60\ \text{Hz})} = 0.0057\ \text{Wb}$$

Q6 - The simple loop is rotating in a uniform magnetic field shown as follows has the following characteristics:

B =1.0 T to the right $\quad$ r=0.1m

l=0.3 m $\quad$ ω_m= 377 rad/s

(a) Calculate the voltage $e_{tot}(t)$ induced in this rotating loop.

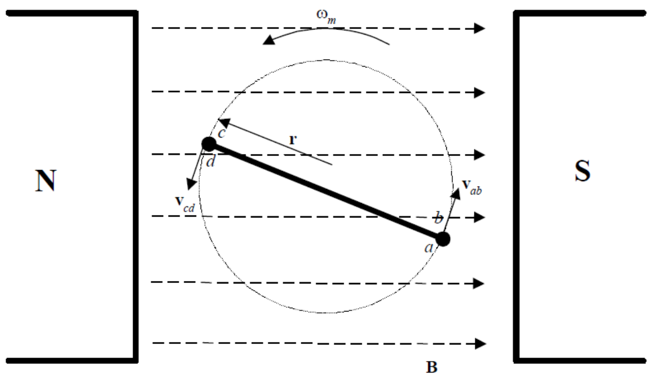
(b) What is the frequency of the voltage produced in this loop?

(c) Suppose that a 10 Ω resistor is connected as a load across the terminals of the loop. Calculate the current that would flow through the resistor.

(d) Calculate the magnitude and direction of the induced torque on the loop for the conditions in (c).

(e) Calculate the instantaneous and average electric power being generated by the loop for the conditions in (c).

(f) Calculate the mechanical power being consumed by the loop for the conditions in (c). How does this number compare to the amount of electric power being generated by the loop?



B is a uniform magnetic field, aligned as shown.

Solution

(a) The induced voltage on a simple rotating loop is given by

$$e_{\text{ind}}(t) = 2r\omega Bl \sin \omega t$$
$$e_{\text{ind}}(t) = 2(0.1 \text{ m})(377 \text{ rad/s})(1.0 \text{ T})(0.3 \text{ m})\sin 377t$$
$$e_{\text{ind}}(t) = 22.6 \sin 377t \text{ V}$$

(b) The angular velocity of the voltage produced in the loop is 377 rad/s. Frequency is related to angular velocity by the equation $\omega = 2\pi f$, so

$$f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60 \text{ Hz}$$

(c) If a 10 Ω resistor is connected as a load across the terminals of the loop, the current flow would be:

$$i(t) = \frac{e_{\text{ind}}}{R} = \frac{22.6 \sin 377t \text{ V}}{10 \ \Omega} = 2.26 \sin 377t \text{ A}$$

(d) The induced torque would be:

$$\tau_{\text{ind}}(t) = 2rilB \sin \theta$$
$$\tau_{\text{ind}}(t) = 2(0.1 \text{ m})(2.26 \sin \omega t \text{ A})(0.3 \text{ m})(1.0 \text{ T}) \sin \omega t$$
$$\tau_{\text{ind}}(t) = 0.136 \sin^2 377t \text{ N}\cdot\text{m}, \qquad \text{clockwise}$$

(e) The instantaneous power generated by the loop is:

$$P(t) = e_{\text{ind}} i = (22.6 \sin 377t \text{ V})(2.26 \sin 377t \text{ A}) = 51.1 \sin^2 377t \text{ W}$$

The average power generated by the loop is

$$P_{\text{ave}} = \frac{1}{T}\int_T 51.1 \sin^2 \omega t \, dt = 25.55 \text{ W}$$

(f) The mechanical power being consumed by the loop is:

$$P = \tau_{\text{ind}}\omega = (0.136 \sin^2 377t \text{ N}\cdot\text{m})(377 \text{ rad/s}) = 51.3 \sin^2 \omega t \text{ W}$$

Note that the amount of mechanical power consumed by the loop is equal to the amount of electrical power created by the loop (within roundoff error). This machine is acting as a generator, converting mechanical power into electrical power.